Looking for correlations in censored data: the Kendall τ test

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Rencontres d'Astrostatistique
14/11/2014

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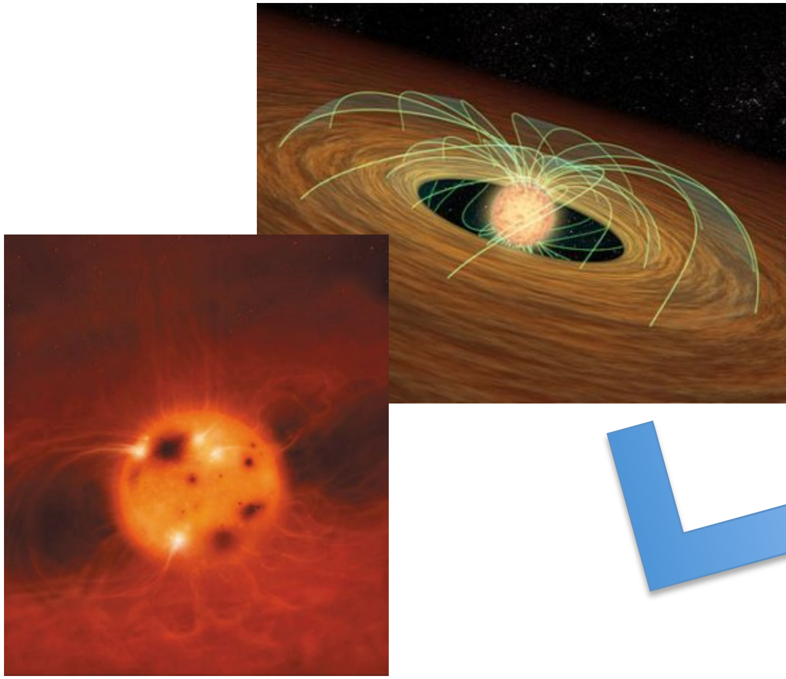
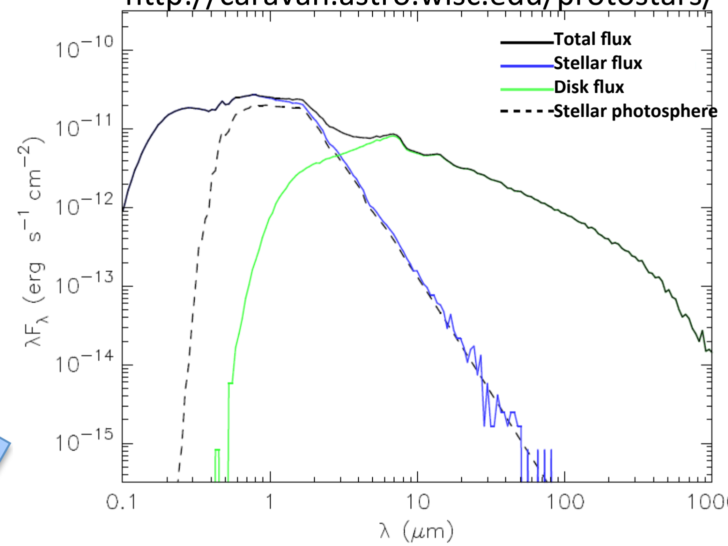
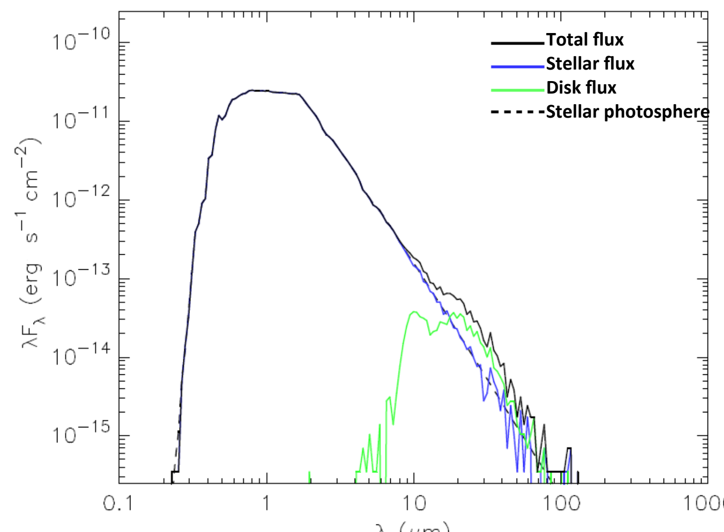
The context



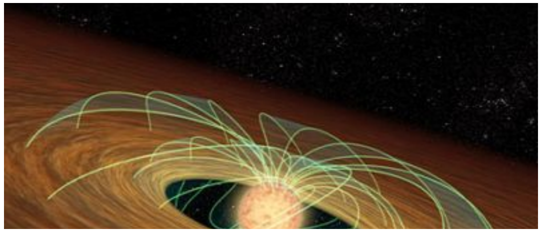
Robitaille et al. 2006, ApJS 167
http://caravan.astro.wisc.edu/protostars/



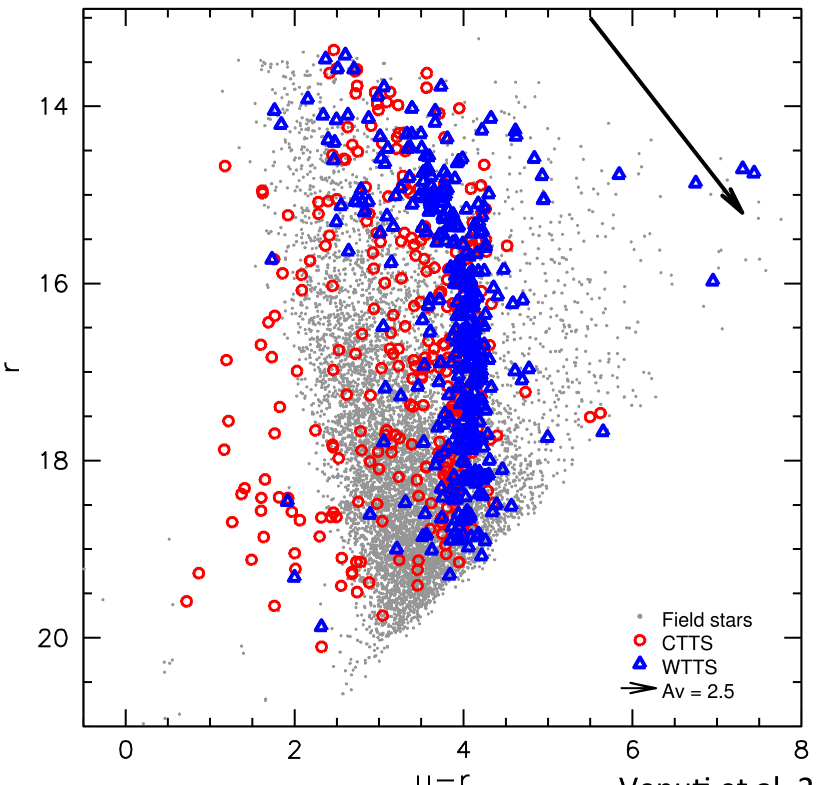


Model of Spectral Energy Distribution (SED) for a young star with (upper panel) and without (lower panel) accretion disk.

The context



u-r colors and r-band magnitudes for accreting (red circles) and non-accreting (blue triangles) members of the star-forming region NGC 2264 (3 Myr old).



Venuti et al. 2014, A&A 570, A82

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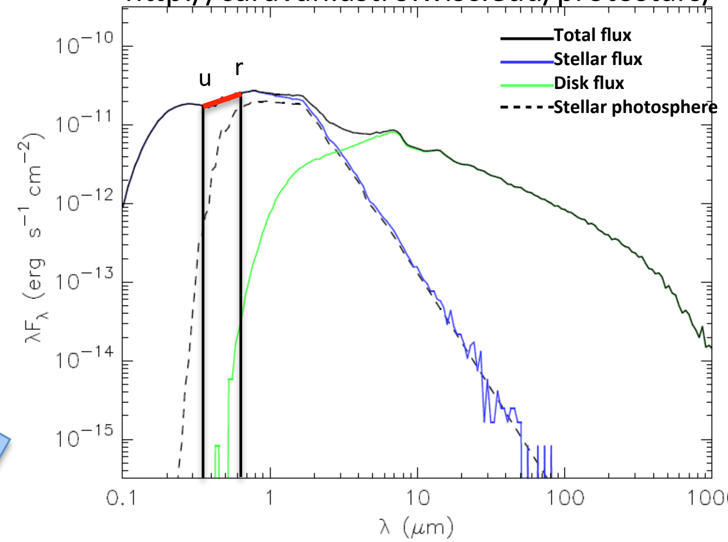
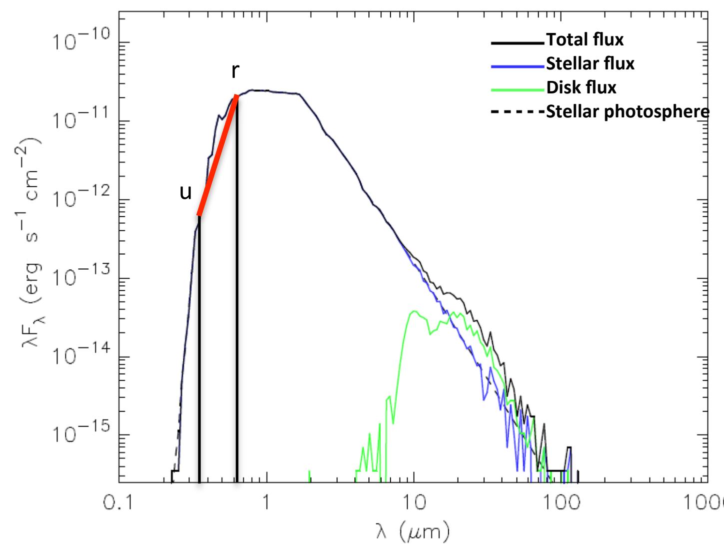
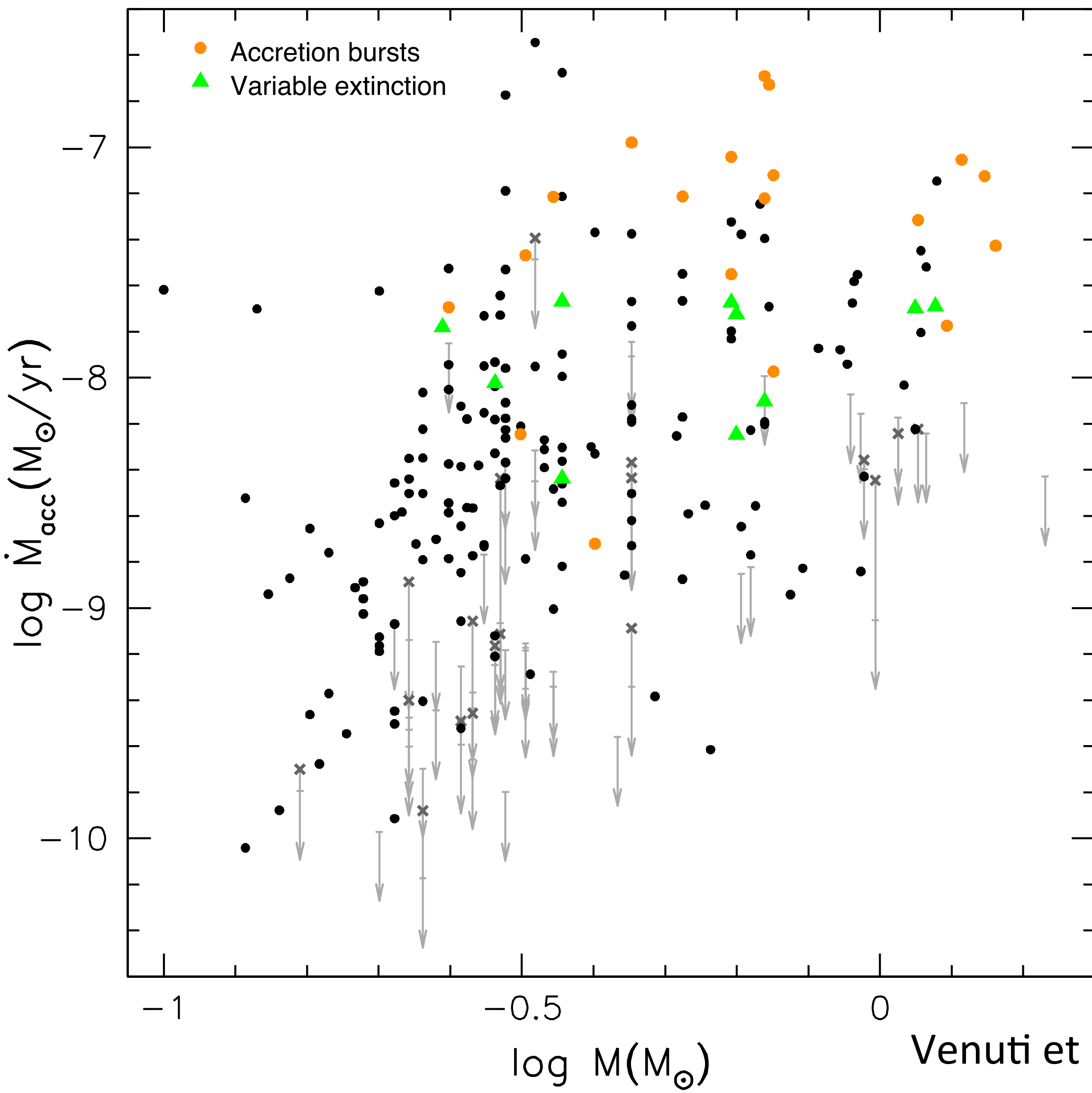




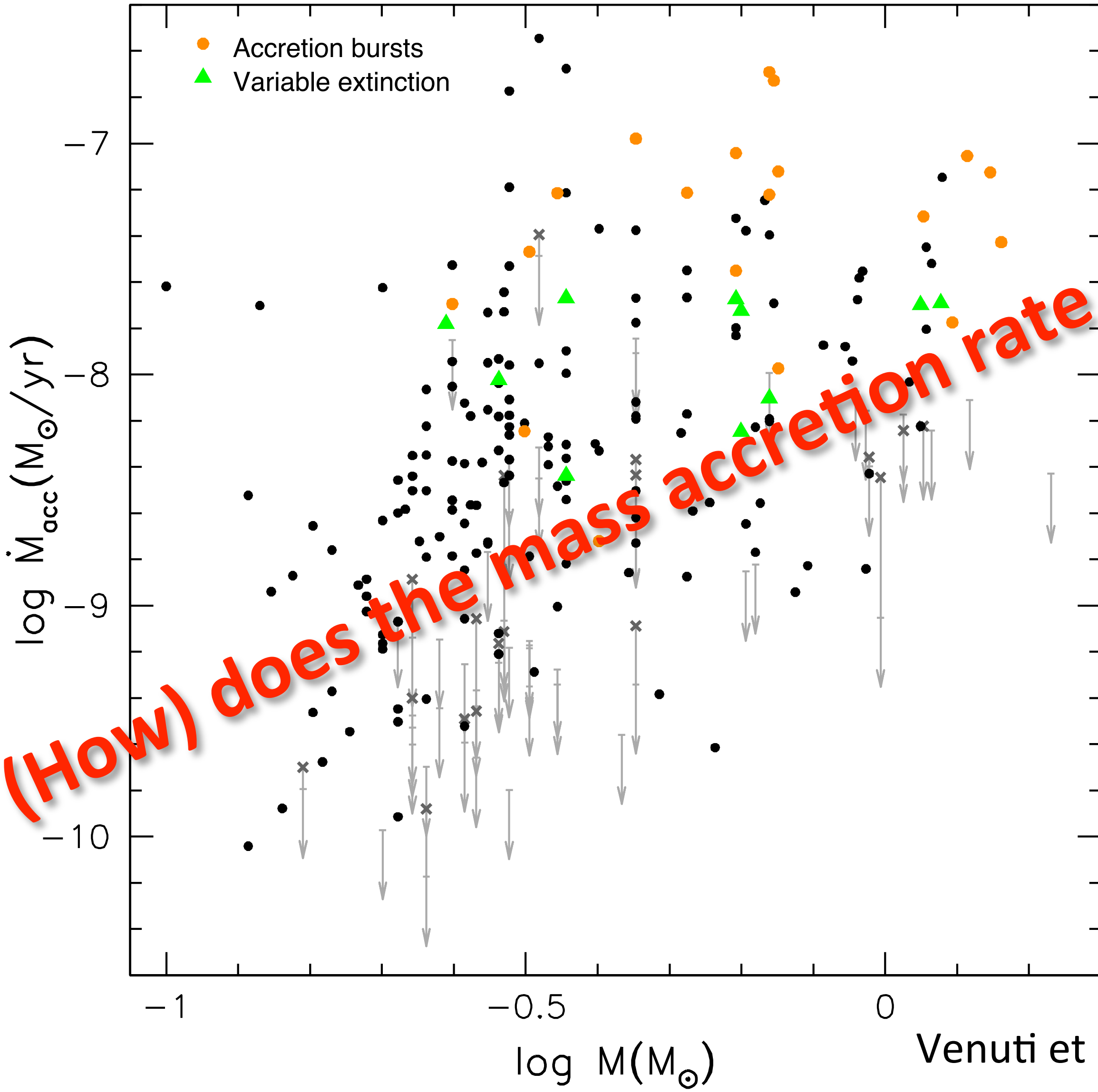
The problem



- 240 objects
- Sample complete in the mass range probed
- 77.5% detections, 22.5% upper limits
- Upper limits are mass-dependent

Venuti et al. 2014, A&A 570, A82

The problem



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The issue

How to properly account for censored data?

- Restrict the statistical analysis to detections
- Take upper limits as actual detections

Least-squares fit

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bypassing the actual information contained in upper limits

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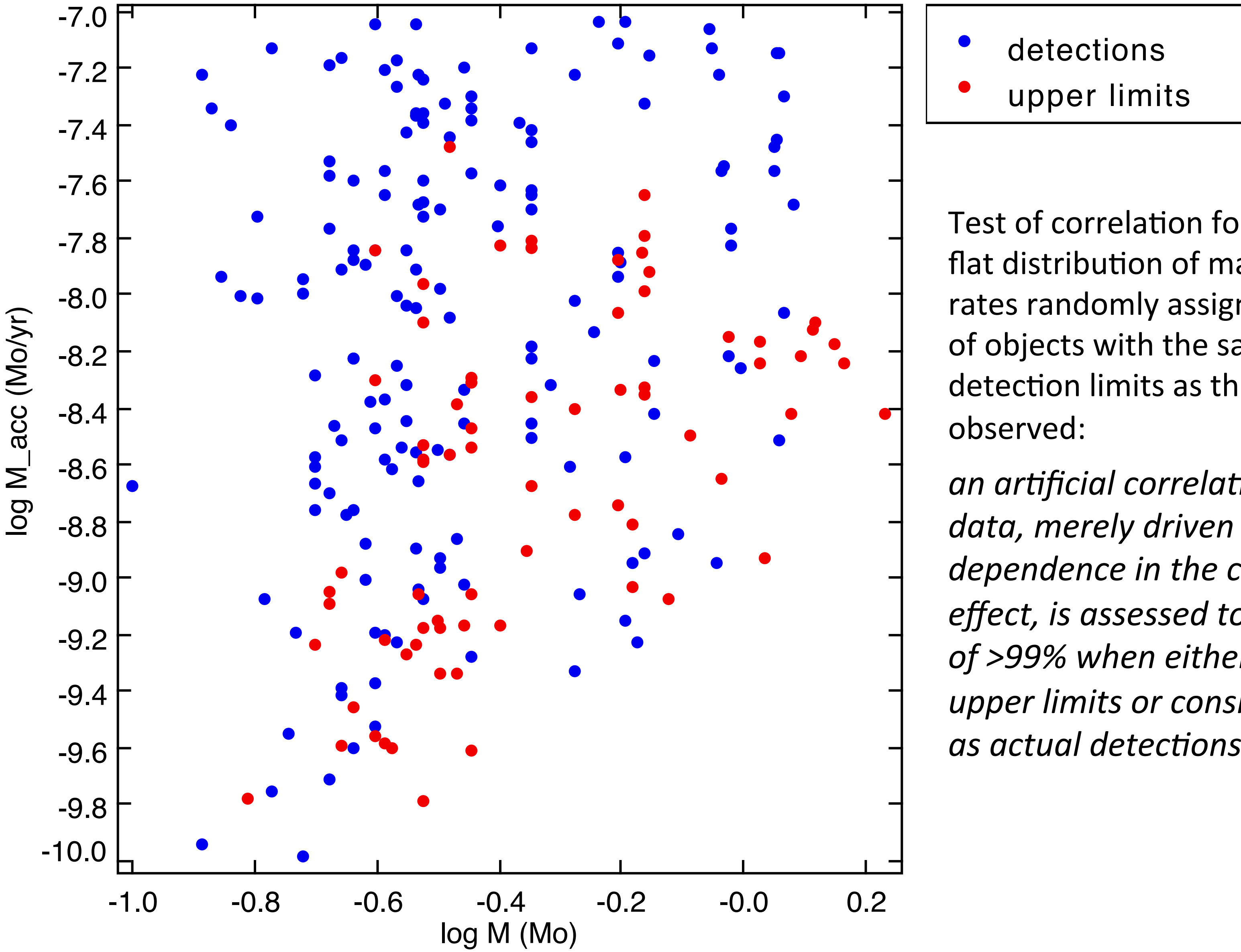

Test of correlation for a synthetic, flat distribution of mass accretion rates randomly assigned to a sample of objects with the same mass and detection limits as the population observed:

an artificial correlation in the data, merely driven by mass dependence in the censoring effect, is assessed to a probability of >99% when either discarding upper limits or considering them as actual detections.

The issue

How to properly account for censored data?

- ~~Restrict the statis nalysis to detections~~
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Least-squares fit

bypassing the actual information contained in upper limits

- Perform the statistical analysis over the whole sample, weighing the different types of information conveyed by actual detections and upper limits

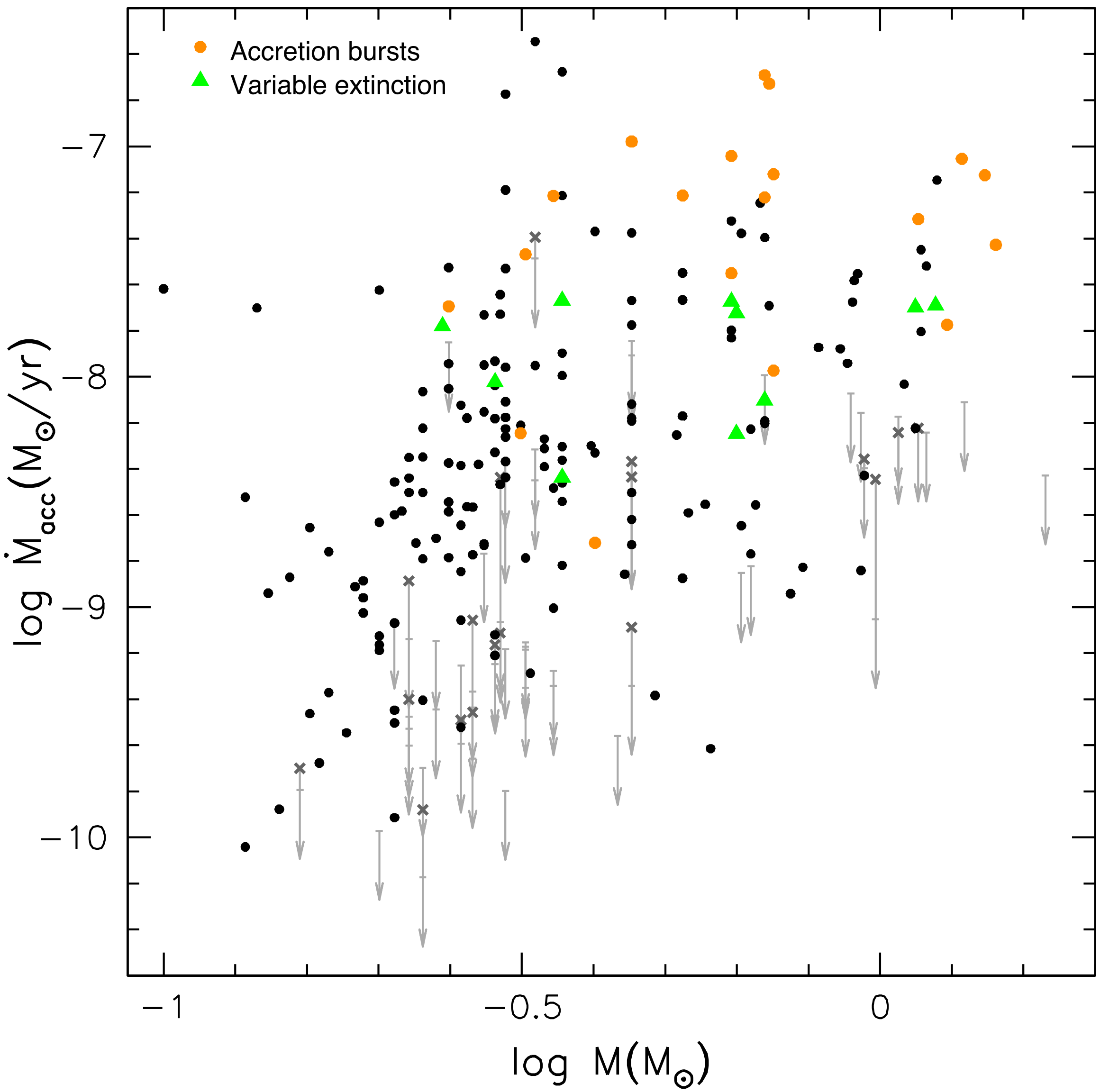
Kendall's τ test for correlation
generalization to the case of censored data

- Feigelson, E. D. & Babu, G. J. 2012, Modern Statistical Methods for Astronomy, Cambridge University Press
- Helsel, D. R. 2012, Statistics for Censored Environmental Data, Wiley

➢ deals with multiple censoring (i.e., both on the x-axis and the y-axis):
  - ties (points at the same abscissa)
  - indeterminate relationships (upper limits)

➢ censored data lower the likelihood of finding a significant correlation, if this is present among the data; they do not "positively" affect the yes/no answer

Kendall's τ test for correlation:
how it works



Correlation: concordant/ discordant variations on x and y

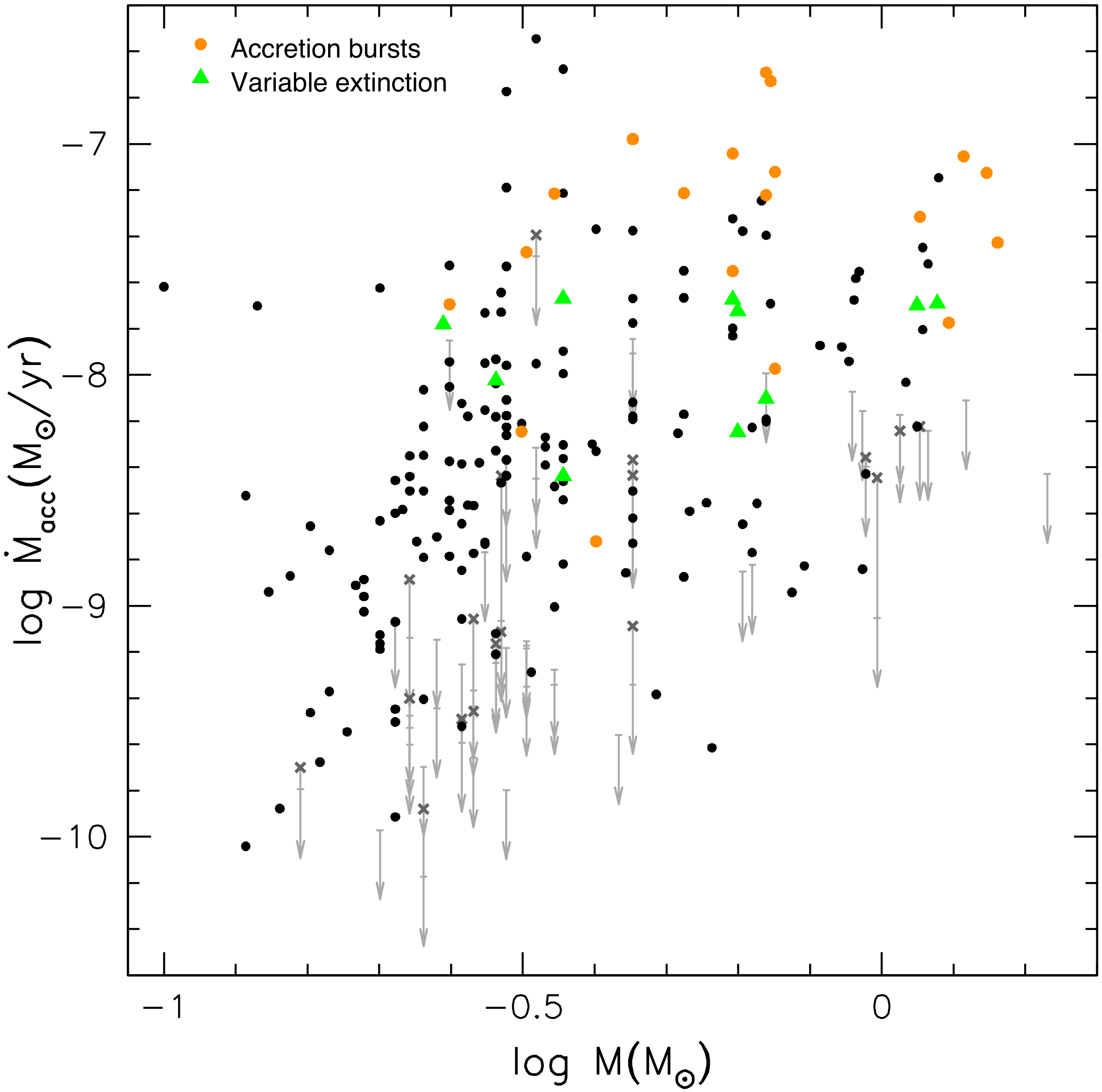
N points

$$\frac{N\,(N-1)}{2} \text{ pairs}$$

How many times do we find a positive/ negative slope?

Kendall's τ test for correlation:

how it works



<u>4 counters:</u>

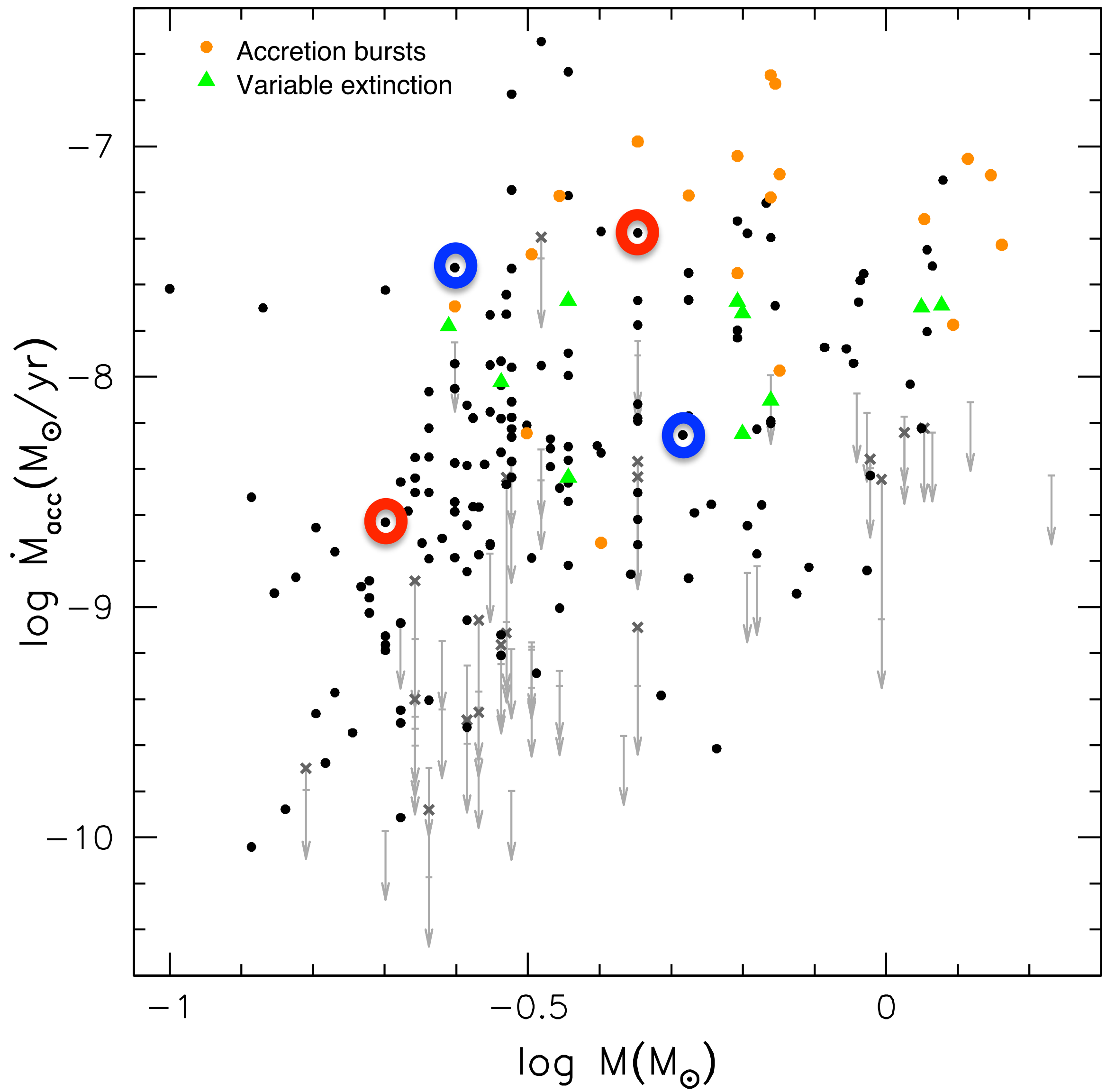
n_C = number of times we measure a positive slope among the sample of pairs

n_D = number of times we find a negative slope

n_{ties_X} = number of times we find a tie on X

n_{ind_Y} = number of times we have an indeterminate relationship due to upper limits

Kendall's τ test for correlation:
how it works



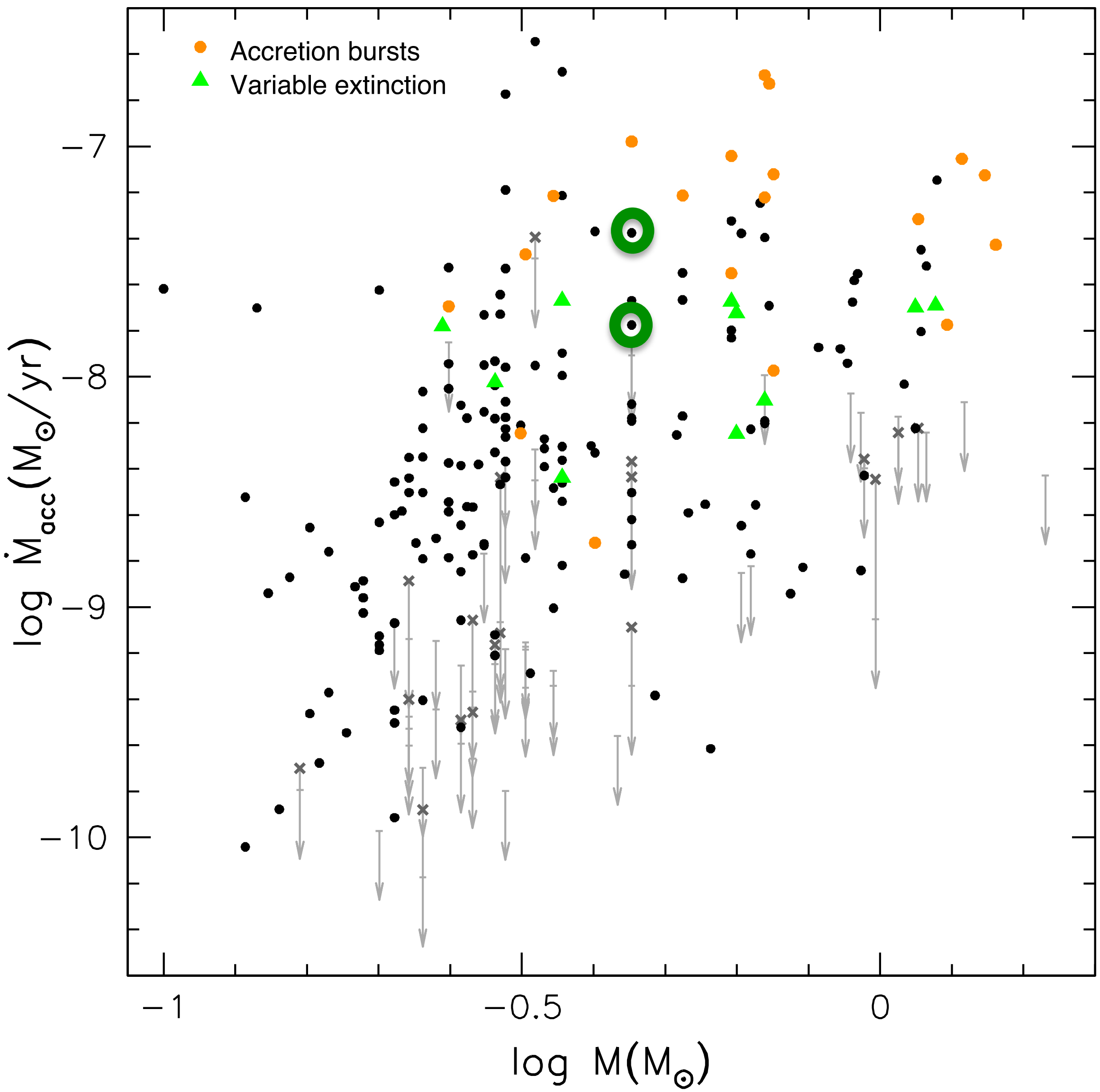
1° case

Pair of detections, $x_1 \neq x_2$

$n_C = n_C + 1$

$n_D = n_D + 1$

Kendall's τ test for correlation:
how it works



1° case

Pair of detections, $x_1 \neq x_2$

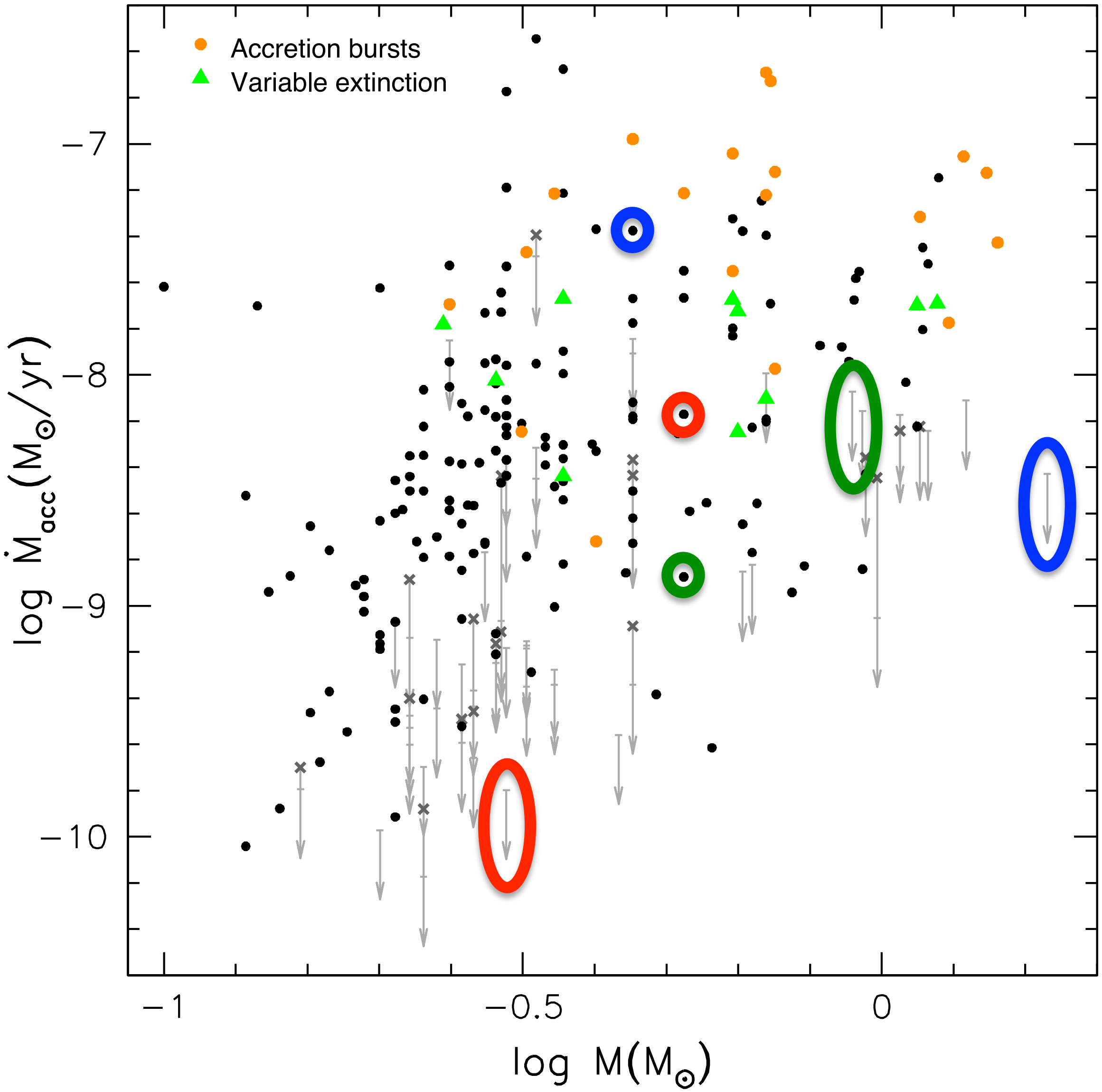
2° case

Pair of points, $x_1 = x_2$

$n_{ties\_X} = n_{ties\_X} + 1$

Kendall's τ test for correlation:

how it works



1° case

Pair of detections, $x_1 \neq x_2$

2° case

Pair of points, $x_1 = x_2$

3° case

1 detect, 1 upper lim, $x_1 \neq x_2$

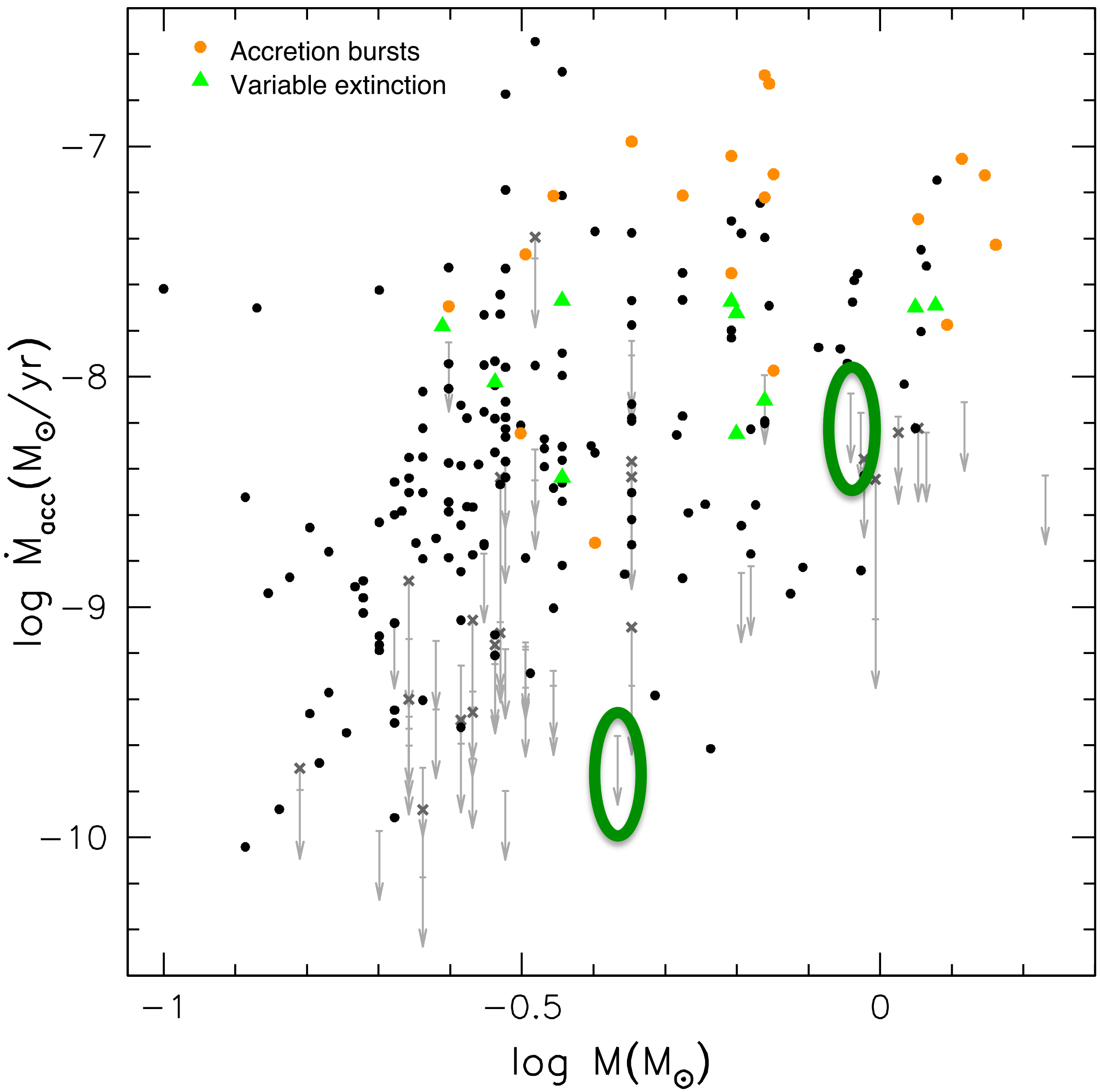
$n_D = n_D + 1$

$n_C = n_C + 1$

$n_{ind\_Y} = n_{ind\_Y} + 1$

Kendall's τ test for correlation:

how it works



1° case

Pair of detections, $x_1 \neq x_2$

2° case

Pair of points, $x_1 = x_2$

3° case

1 detect, 1 upper lim, $x_1 \neq x_2$

4° case

Pair of upper limits, $x_1 \neq x_2$

$n_{ind\_Y} = n_{ind\_Y} + 1$

Kendall's τ test for correlation:
how it works

Correlation coefficient: $\boldsymbol{\tau = \dfrac{n_C - n_D}{n_{tot}}}$

where n_{tot} is the total number of pairs:

- $n_{tot} = 0.5\ N\ (N-1)$

  or

- $n_{tot} = \mathrm{sqrt}[\ (0.5\ N\ (N-1) - n_{ties\_X})*(0.5\ N\ (N-1) - n_{ind\_Y})\ ]$

Kendall's τ test for correlation:
how it works

In the null hypothesis of no correlation, τ follows the normal distribution centered on zero and with variance

$$\sigma^2 = \frac{2\,(2\,N + 5)}{9\,N\,(N - 1)}$$

(to be adapted to take into account different groups of ties)

$\mathbf{Z = \tau / \sigma}$ expresses the significance of the correlation result

The results

First question: does M_{acc} correlate with M_*?

Yes (τ = 0.28, σ = 0.04, z > 6)

Tested against purposedly generated flat distributions of M_{acc} values in the same M_{acc} range and with the same upper limits distribution – no correlation is statistically detected in these cases

Second question: what slope?

Akritas-Theil-Sen non-parametric regression (Feigelson & Babu 2012):

- Take a first guess for the slope and explore a range around this value
- For each test slope, subtract the Y=mX trend from the observed distribution and repeat the Kendall's τ test
- The best value of slope m is the one that, subtracted to the distribution, produces τ = 0, while the m values range corresponding to $\tau = \pm n\sigma$ will provide an $n\sigma$ error bar on the slope

For the case in exam, a slope of 1.5 +/- 0.2 is obtained.